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FLUCTUATION ORIGIN OF COSMIC RADIATION***

by

J. R. Wayland

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FLUCTUATION ORIGIN OF COSMIC RADIATION*

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ABSTRACT

We have investigated what conditions are placed upon a source of cosmic ray particles within a model of the origin of cosmic rays in which fluctuations in the momentum-changing process are important. This is done by considering the relationship between $\langle \Delta p \rangle$ and $\langle \Delta p^2 \rangle$, where Δp is the momentum change in a collision. With the assumption that the energy due to turbulent motion, cosmic rays, and the magnetic field are linearly related, we find an expression for the number density of cosmic ray particles ejected from a changing volume in terms of their average kinetic energy. We consider this in terms of the fluctuation model to obtain other source requirements. It is found that over a large range of astrophysical phenomena the dominance of fluctuation can be important to the origin of cosmic radiation.

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I. INTRODUCTION

Recent work⁽¹⁾⁽²⁾ has shown that it is possible to explain the primary cosmic ray spectrum in terms of a process in which statistical fluctuations dominate. L. Davis⁽³⁾ had previously investigated the influence of fluctuations on the observed spectra. His approach was to consider the possibilities of statistical fluctuations in the interactions of cosmic rays with varying galactic magnetic fields. He, however, did not consider the case of dominating deceleration.

Previously it was thought that fluctuations could not be significant in the origin of cosmic radiation⁽⁵⁾. This evaluation was based upon the assumption that acceleration processes dominate. If, however, deceleration effects are more important than acceleration, then we have the case where fluctuations can explain the high energy spectrum.

We expect the source requirements that such a model would impose to be dissimilar to those of a model in which acceleration is the prevailing influence. Accordingly, we have investigated what conditions are placed upon a source of cosmic rays within the fluctuation origin theory.

In Section II we will outline the theory and give the germane results. Section III will be devoted to the consequences of assuming that the acceleration - deceleration coefficient, a , and the fluctuation coefficient, b , are related by $a = kb$. With the assumption that the energy due to turbulent motion, cosmic rays and the magnetic fields are linearly related we find an expression for the number density of cosmic ray particles ejected from changing volume in terms of their average kinetic energy (Section IV). This result is then considered in terms of the fluctuation model to check for further source requirements in Section V.

II. THE FLUCTUATION ORIGIN OF COSMIC RAYS

Let us consider the sudden injection of particles into a region of moving plasma centers that act as scattering centers. We assume that the injection is over a short enough time that we can write this as a delta function. Thus we are also requiring that there is only one injection. By treating this case in detail we can then use our results for all such injections by a proper averaging procedure. We will also assume that the particles are all injected with the same momentum, p_0 . We would obtain the same type of results for a power law injection in which p_0 is the minimum injection momentum and the maximum injection momentum is not a great deal larger than p_0 .

The injected particles in the turbulent region (I) will undergo scatterings that change both their momentum and direction, i.e., they will diffuse in both space and momentum. We will assume that the spatial diffusion coefficient, D , is independent of the spatial and momentum coordinates. [This means that we can not apply our results to the low energy particles in our solar system, but should apply them to the higher energy "galactic" cosmic radiation.] The diffusion in momentum space will be described by a Markoff process. Thus we have assumed that at each momentum "scattering", the particle is just at the point of forgetting what has happened in the last "scattering". In other words, the conditional probability depends only on the value of the momentum at the time of the previous "scattering". The momentum diffusion will then be described by a Fokker-Planck Eqn.

When the particles diffuse to the boundary of the turbulent region (I) they escape into interstellar space region (II). This escape is caused mainly by "radiation" across the boundary. Then the flux will be proportional to the difference of particle densities in the two mediums. We note that for particles in the turbulent region the particle density in interstellar

space can, to a first approximation, be considered to be zero.

In interstellar space, region II, we will assume that the turbulence is much less than in the turbulent region I. Thus the acceleration will be small in comparison to that experienced in region I. We will approximate this by postulating that the particles are no longer experiencing an acceleration process. The particles will, however, diffuse spatially, as is indicated by the almost complete isotropy of cosmic radiation. How the particles will propagate from the source to the point of observation is still an open question. They may move along kinked magnetic lines of force. This can result in a streaming or even in what appears to be diffusion depending upon the characteristics of the magnetic field. Regardless of the true mode of propagation we will assume the simple case of diffusion. This will allow us to investigate the effects of fluctuations in the acceleration process without treating in detail the propagation process.

When the particles reach the boundary of interstellar space (region II) with intergalactic space we will assume that they freely "radiate" into the intergalactic medium. For an observer at earth, this boundary is very far away. We would expect that to a good approximation we can treat region II as spherical. In this model the density of cosmic rays in intergalactic space is much smaller than the density in interstellar space. This will impose the condition that the particle density, n , should decrease as one approaches this boundary (and we will assume that it approaches zero).

To give an example of this type of model consider the explosion of a supernovae. As long as the particles are accelerated dominately within the expanding volume of turbulent ejecta we can apply this model. It may also be possible that there are reoccurring implosions that will act as injection sources⁽⁴⁾.

The problem is solved for a time-dependent state. However, the cosmic

ray intensity appears to be constant with time. Thus we can use the relatively easily obtained time dependent solution and integrate it over all past time to find its total contribution to the present. In a forthcoming paper we will discuss this point in greater detail.

The problem can be written as

$$(1) \quad \frac{\partial n_1}{\partial t} - D_1 \nabla^2 n_1 + \frac{\partial}{\partial p} \left(\frac{\langle \Delta p \rangle}{\Delta t} n_1 \right) - \frac{1}{2} \frac{\partial^2}{\partial p^2} \left(\frac{\langle \Delta p^2 \rangle}{\Delta t} n_1 \right) + \frac{n_1}{T_1} = 0,$$

for $t > t_0$ in region I and

$$\frac{\partial n_2}{\partial t} - D_2 \nabla^2 n_2 + \frac{n_2}{T_2} = 0,$$

for $t > t_0$ in region II. Here n is the particle density, D is the diffusion coefficient, $\langle \Delta p \rangle$ and $\langle \Delta p^2 \rangle$ the first and second moments of the momentum changing process, and T is the effective lifetime against removal by interaction processes within the medium. At the boundary of region I and II, $r = a_s$, we require

$$(3) \quad \frac{\partial n_1}{\partial r} = h(n_1 - n_2),$$

$$(4) \quad D_1 \frac{\partial n_1}{\partial r} = D_2 \frac{\partial n_2}{\partial r},$$

and at the boundary of region II (interstellar space) with intergalactic space, $r = R$,

$$(5) \quad n_2 \rightarrow 0.$$

Note that h determines the confinement conditions of particles in region I. As $h \rightarrow 0$ we have the case of free escape across the boundary, and as $h \rightarrow \infty$ we have complete confinement, i.e., there is no flux across the boundary.

The initial condition at time $t = t_0$ is

$$(6) \quad n = q_0 \delta(t - t_0) \delta(\rho - \rho_0) \delta(r - r_0),$$

where r_0 is the radius of the "point" source. (We will consider the case where $r_0 \rightarrow 0$.) If we assume that

$$(7) \quad \frac{\langle \Delta \rho \rangle}{\Delta t} = a \rho,$$

and

$$(8) \quad \frac{\langle (\Delta \rho)^2 \rangle}{\Delta t} = 2b \rho^2,$$

we find that the solution in region II is given by (see appendix)

$$(9) \quad \gamma_2 = \frac{3.88 q_0 r_0^{1/2} \sqrt{h} \rho_0 A}{h \rho_0 b^{1/2} a_s^3 r} \left(\frac{\rho}{\rho_0} \right)^{\gamma},$$

where $\gamma_0 = \frac{2b}{a(a-b)},$

$$A = - \left(\frac{1}{a_s} - \frac{h D_1}{D_2} \right) (r - a_s) - \frac{9.6 D_1 \gamma_0}{a_s^2} - \frac{\gamma_0}{T_1} - \frac{(a-b)^2}{4b^2},$$

and $\gamma = \frac{a(a-b)}{8b^2} \ln(\rho/\rho_0) + \frac{3b-a}{2b}.$

This is an asymptotic solution that is valid when $\ln(P/P_0) > 1$.

Note that we can write Equation (9) in the form

$$(10) \quad \gamma_2 = \text{constant} \left(\frac{P_0}{P} \right)^\gamma.$$

When one applies this to the primary cosmic ray spectrum, it is found that only when there is a steady decrease in the mean statistical momentum change, $\langle \Delta P \rangle$, can one fit the observed spectrum. Then deceleration is dominating over acceleration. The particles that are observed at very high energies are the result of a series of favorable acceleration scatterings. This is just the result of fluctuations in the acceleration process. But what are the conditions under which we can expect this type of model to apply? We will investigate this question in the next sections.

III. SOURCE REQUIREMENTS FROM $a = kb$

If we assume that $a = kb$, we can obtain the observed primary cosmic ray spectrum. We can vary k and P_0 in the fitting procedure for the correct slope of the power law momentum spectrum. As one would expect, there is a coupling between k and P_0 . This is illustrated by the fact that for $P_0 = 0.1$ GeV/c, $k = -0.28 \pm 0.02$; for $P_0 = 1.0$ GeV/c, $k = -0.32 \pm 0.02$; and for $P_0 = 10.0$ GeV/c, $k = -0.37 \pm 0.02$, etc. We note that as P_0 varies over a rather large range that k remains at about -0.3 or -0.4 . Let us, for the present, disregard P_0 and investigate what conditions we can place upon the source by the requirement of $a = kb$.

We have shown in a previous paper⁽²⁾ that one can write for the first and second moment

$$(11) \quad \Delta \bar{a} = \frac{8}{3} \bar{B}^2 - \frac{2 B_e \lambda}{R} + \frac{2 B_e \lambda \bar{B}^2}{3 R},$$

$$(12) \quad \Delta \bar{b} = \frac{B_e^2 \lambda^2}{R^2} - \frac{4 B_e \lambda \bar{B}^2}{R} + \frac{4}{3} \bar{B}^2,$$

where $\bar{B}^2 = \int B^2 f(V) dV.$

In the above expressions $B_e = V$ = velocity of the scattering centers, $f(V)$ = velocity distribution of the scattering centers, $B_e c = V_e$ = velocity of expansion, R = radius of expansion, and λ = mean free path between scatterings. We have assumed that the scattering centers are receding

from each other as the result of spherical expansions from a common center. The amount of momentum change at each scattering is calculated from Fermi theory. It is also assumed that the scattering is essentially isotropic. Terms of order greater than \bar{B}^3 have been ignored.

If we include Equations (11) and (12) in $a = kb$ and solve for \bar{B} we find

$$(13) \quad \bar{B} = \left[\frac{2 + \frac{k B_e \lambda}{R}}{\frac{4R}{3 B_e \lambda} (2-k) + 2(1+2k)} \right]^{1/2}.$$

It is very troublesome to attempt to estimate λ . However, we can give a physical meaning to the ratio R/λ under suitable conditions. If the mean size of the scattering centers is approximately equal to the distance between centers, we note that

N_{sc} = number of scattering centers,

$$(14) \quad N_{sc} \approx \frac{\frac{4}{3} \pi R^3}{\frac{4}{3} \pi \lambda^3} = \left(\frac{R}{\lambda} \right)^3.$$

Even if the radius of the scattering centers, R_{sc} , is $\lambda/10$, then $N_{sc} = 10^3 (R/\lambda)^3$. We can plot \bar{B} as a function of R/λ for various B_e at a fixed k . The results for $k = -0.30$ and $R_{sc} = \lambda$ are shown in Fig. I.

Note that for a fixed B_e as the number of scattering centers increases the average velocity of the scattering centers decreases. This is what one would expect, as there would then be more of a chance for scattering, and hence for fluctuations to occur. Also note that for a fixed R/λ the required \bar{B} increases for increasing B_e . One can understand this by remembering that in our case deceleration is dominant.

The larger the expansion rate, i.e., B_e , the more efficient is the deceleration, and hence the larger \bar{B} must be to allow for the few favorable sequences of scatterings that give rise to the fluctuations. When $R_{sc} < \lambda$ the conditions for fluctuations are increased because the number of scattering centers are increased.

To what astrophysical phenomena can we apply these considerations? Almost any ionized gas that is in a turbulent state appears to be able to produce energetic particles. If there is a reason for deceleration to dominate over acceleration, one can perhaps have the fluctuations required. One of the simpler cases is that of a region undergoing expansion. This is not the only case, but it is common. As an example, consider novae and supernovae. Here B_e is about 3×10^{-3} and 2×10^{-2} respectively⁽⁶⁾. Then for $R_{sc} \sim \lambda/10$ and $N_{sc} \sim 10^9$ this implies that \bar{B} is 10^{-3} to 10^{-2} , a reasonable requirement.

IV. THE RELATION BETWEEN THE AVERAGE KINETIC ENERGY OF PARTICLES EJECTED FROM A TURBULENT REGION AND THE TOTAL ENERGY.

One of the more surprising results of the application of the equipartition of energy is the prediction of the cosmic ray spectrum in the form of a power law by Syrovatsky⁽⁷⁾. Later this work was extended to include a dependence upon a changing volume by Satô⁽⁸⁾. In both of these works, the assumption is made that the average kinetic energy of the particles can be replaced by its total energy. This is a rather problematical step. Below, we will obtain the number of particles ejected as a function of its average kinetic energy without assuming the equipartition of energy. We will not assume that we can replace the average kinetic energy by an energy, but will compare the average kinetic energy with results from the fluctuation origin of cosmic rays to try to obtain information about conditions imposed upon the source and the turbulent region.

We will consider high-energy cosmic rays which are produced within a region of turbulent motion of a magnetized plasma. The energy within the region will have terms from thermal motion, turbulent motion, magnetic field, cosmic ray particles, radioactive nuclei, etc. We shall assume that the only modes to offer significant contribution to the total energy are the turbulent motion, the magnetic field and the cosmic ray particles. Then we can write

$$(15) \quad E = E_{\text{turb.}} + E_{\text{mag.}} + E_{\text{cr.}}$$

We also note that the cosmic ray radiation from the region is given by

$$(16) \quad -\dot{I}_{cr} = \frac{dE}{dt} + P \frac{dV}{dt},$$

where P is the total pressure.

When the particles are ejected we will assume that

$$(17a) \quad E_{turb} = a_1 E_{cr}$$

$$(17b) \quad E_{mag} = a_2 E_{cr}$$

so that we can write

$$(18) \quad E = (1 + a_1 + a_2) E_{cr} = (1 + a_1 + a_2) \bar{T} N_o$$

where \bar{T} is the average kinetic energy per particle and N_o is the total number of particles. We note that

$$(19) \quad -\dot{I}_{cr} = \bar{T} \frac{dN_o}{dt}.$$

In the above model of the fluctuation origin of cosmic rays, it is assumed that particles are decelerated. One way of doing this is to allow for a changing volume. Let us then take

$$(20) \quad E = \text{constant } V^q$$

With the aid of Equation (20), we can combine Equations (16), (18) and (19) to find

$$(21) \quad \bar{T} \frac{dN_o}{dt} = (1 + a_1 + a_2) \left(1 + \frac{q}{3}\right) \frac{dE_a}{dt},$$

where $\beta_i = P_i V / E_i$ and $P_i = -(dE_i / dV)_S$ = partial pressure of the i^{th} mode ($\beta = \frac{1}{3} \sum_{i=1}^3 \beta_i$). Recalling that $E_{\text{cr}} = \bar{T} N_0$, we can solve Equation (21) to obtain

$$(22) \quad N_0 = k_1 \bar{T}^{-\alpha_1},$$

where

$$\alpha_1 = \frac{(1+a_1+a_2)(1+\frac{\beta}{q})}{(1+a_1+a_2)\frac{\beta}{q} + a_1 + a_2}$$

$$= 1 + \frac{1}{a_1+a_2 + (1+a_1+a_2)\frac{\beta}{q}} \equiv 1 + \frac{1}{\alpha_2}.$$

Satô⁽⁶⁾ shows that $\beta = 4/9$.

Within the framework of our model, we need to know how \bar{T} , E and V are changing with time. To do this, we must find \bar{T} , E and V as a function of N_0 and q . A little algebra gives

$$(23) \quad \bar{T} \propto N_0^{-\frac{4(1+a_1+a_2) + 9(a_1+a_2)q}{(1+a_1+a_2)(9q+4)}}$$

$$(24) \quad E \propto N_0^{-\frac{9q}{(1+a_1+a_2)(9q+4)}}$$

$$(25) \quad V \propto N_0^{-\frac{9}{(1+a_1+a_2)(9q+4)}}$$

If we indicate that a quantity is increasing by \uparrow and decreasing by \downarrow we can make the following table:

q	$\frac{-4(1+a_1+a_2)}{9(a_1+a_2)}$	-4/9	0
\bar{T}	↑	↓	↑
E	↓	↑	↓
V	↑	↓	

Table I: The Time Variation of \bar{T} , E and V.

In the fluctuation origin of cosmic rays we require that V is increasing (i.e., $q < -4/9$), and that E and \bar{T} are decreasing. This last condition is the result of deceleration dominating. Then we have that

$$-\frac{4(1+a_1+a_2)}{9(a_1+a_2)} < q < -\frac{4}{9}.$$

If we have equipartition of energy, i.e., $a_1 = a_2 = 1$, then $-2/3 < q < -4/9$.

However, if a_1 or $a_2 \gg 1$ and a_2 or $a_1 \approx 1$, q is restricted to a value very close to $-4/9$. This places severe restrictions upon the fluctuation model.

If a_1 or $a_2 = 0$ and a_2 or $a_1 = 1$, then

$$-\frac{8}{9} < q < -\frac{4}{9}$$

and we have a wider range on q. In the case of a_1 or $a_2 = 0$ and a_2 or $a_1 \ll 1$

there is an even wider range on q. We note that if we have either the turbulent or the magnetic energy dominating, that the probability of cosmic ray particles' being produced forces very restrictive requirements upon a source. Whereas if either the turbulent or magnetic mode is absent, the range of q becomes much greater. We can, from the above analysis, write

$$(26) \quad \frac{dV}{dt} = \frac{\frac{1}{a_1}(1+a_1+a_2)V}{qE} \frac{dE_{turb}}{dt}.$$

As an approximation, set $E_{\text{turb}} = 1/2 m \bar{v}_{\text{turb}}^2$, where m is the mass of the turbulent medium and \bar{v}_{turb} is the average velocity of the turbulent centers. Note that $\bar{v}_{\text{turb}}/c = \bar{B}$. Here $dv/dt > 0$ and $dE_{\text{turb}}/dt < 0$, which implies that $q < 0$. Also, we must have turbulent motion as $a_1 \neq 0$. Combining Equation (26) with $dV/dt = 4\pi R^2 c B_e$ we find

$$(27) \quad q = \frac{\frac{1}{a_1} (1 + a_1 + a_2) V \bar{B} m}{4\pi R^2 B_e} \frac{dN_{\text{turb}}}{dt}.$$

As an example, if we consider a supernova remnant with $dv_{\text{turb}}/dt \sim -.1 \text{ cm/sec}^2$, $R \sim 1 \text{ l.y.}$, $m \sim .1 M_{\odot}$, $B_e \sim 10^{-3}$, $\bar{B} \sim 10^{-2}$ and $W \sim 10^{50}$ ergs, we find that $-q \sim 0$ (1). This very crude estimate shows that our values of q are within reason.

V. SOURCE REQUIREMENTS FROM $N_o = k_1 \bar{T}^{-\alpha_1}$

We will investigate what source requirements result from combining the results of Sections II and IV. If we use Equation (10) we find that

$$(28) \quad N_o = \frac{k_2}{(\gamma-1)} \left(\frac{p_o}{p_m} \right)^{\gamma-1},$$

$$(29) \quad \langle p \rangle = \frac{k_2}{(\gamma-2)} \left(\frac{p_o}{p_m} \right)^{\gamma-1} p_m,$$

$$(30) \quad \langle E \rangle = \frac{k_2}{(\gamma-2)} \left(\frac{p_o}{p_m} \right)^{\gamma-1} p_m \left[1 + \frac{(\gamma-2)^2 m^2}{k_2^2 p_m^2} \left(\frac{p_o}{p_m} \right)^{2(1-\gamma)} \right]^{\frac{1}{2}},$$

$$(31) \quad \langle T \rangle = \frac{k_2}{(\gamma-2)} \left(\frac{p_o}{p_m} \right)^{\gamma-1} p_m \left\{ \left[1 + \frac{(\gamma-2)^2 m^2}{k_2^2 p_m^2} \left(\frac{p_o}{p_m} \right)^{2(1-\gamma)} \right]^{\frac{1}{2}} - \frac{m(\gamma-2)}{k_2 p_m} \left(\frac{p_o}{p_m} \right)^{1-\gamma} \right\}$$

where k_2 is a constant, p_m is the minimum momentum considered and $\langle p \rangle$, $\langle E \rangle$, $\langle T \rangle$ are the average momentum, total energy and kinetic energy of all cosmic ray particles, respectively, and m is the mass of the particle considered. We know that $\bar{T} = \langle T \rangle / N_o$ which, when combined with $N_o = k_1 \bar{T}^{-\alpha_1}$, can be written as

$$(32) \quad \langle T \rangle^{\alpha_2} = N_o k_1^{\alpha_2},$$

where $\alpha_3 = \alpha_1 \alpha_2$.

If we insert Equations (28) and (31) into Equation (32) and rearrange, we obtain an equation for γ

$$(33) \quad \chi^{\alpha_3} - k_3 \chi - k_3 = 0,$$

where

$$k_3 = \left(\frac{k_2}{k_1}\right)^{\alpha_2} P_0^{\alpha_3} \left\{ \left[1 + \frac{\chi^2 m^2}{k_2^2 P_m^2} \left(\frac{P_0}{P_m}\right)^{-2(\chi+1)} \right]^{\frac{1}{2}} - \frac{m \chi}{k_2 P_m} \left(\frac{P_0}{P_m}\right)^{-(\chi+1)} \right\}$$

Thus we have an equation for γ as a function of the experimental parameters $k_1, k_2, P_0, P_m, a_1, a_2, m$. We can simplify this by noting that

$$(34) \quad k_1 = k_2 \frac{T^{\alpha_1}}{(\chi+1)} \left(\frac{P_0}{P_m}\right)^{-(\chi+1)}$$

The parameter k_2 can be found from the observed spectrum of cosmic rays. We will use the results quoted by Webber.⁽⁷⁾ Equation (33) is then solved numerically by a modified Newton-Ralphson method for various fixed values of P_0, P_m, a_1 and a_2 . Some representative results are shown in Figs. II to VII. (Here we have assumed m is the mass of the proton,)

The value of γ varies between 2 and 3, increasing with increasing momentum in the primary cosmic ray spectrum.⁽²⁾ The average kinetic energy of cosmic ray particles is of the order of unity.⁽⁴⁾ Let us consider 5 cases: (1) where equipartition of energy exists ($a_1 = a_2 = 1$); (2) where the turbulent (magnetic) energy is twice the cosmic ray energy and the magnetic (turbulent) and cosmic ray energies are the same (a_1 or $2 = 2, a_2$ or $1 = 1$); (3) where $E_{\text{turb (mag.)}} = 10 E_{\text{cr}}$ and

$E_{\text{mag. (turb.)}} = E_{\text{cr}}$ (a_1 or $2 = 10$, a_2 or $1 = 1$); (4) where $E_{\text{turb.(mag.)}} = 30 E_{\text{cr}}$ and $E_{\text{mag.(turb.)}} = E_{\text{cr}}$ (a_1 or $2 = 30$, a_2 or $1 = 1$); (5) where $E_{\text{turb.}} = 0.1 E_{\text{cr}}$ and $E_{\text{turb.}} = E_{\text{mag.}}$ (a_1 or $2 = a_2$ or $1 = 0.1$). This, of course, does not exhaust all of the possibilities. However, it will be sufficient to outline the main characteristics of the source requirements that we seek.

The results of solving Equation (33) for each case are shown in Figs. II, III, IV, V and VI. We have not shown roots that are less than 2. A number of general features can be seen from an inspection of these results. We first note that generally when $P_o = P_m = 0.1 \text{ GeV/c}$ the values of γ are too low to be of any real contribution. It is only when $P_o \sim 1 \text{ GeV/c}$ or greater that acceptable roots appear. However if we go much above 5 GeV/c the values of γ are rather large. Note that it is possible to find the same curve at different values of q by a suitable choice of P_o and P_m (see curve IV, Fig II). The greater the value of $|q|$, the stronger is the volume expansion, and hence the deceleration. This seems to imply that a "balance" of P_o , the injection momentum, against the deceleration is necessary. As we increase the energy in the turbulent (or magnetic) mode over the other two modes, we find that there is a greater variation of γ with a smaller P_o change. One would expect this if the turbulent region were trying to establish an equipartition of energy. On the other extreme, when the cosmic ray mode overpowers the other two modes (Fig. VI), we find that it is very difficult to obtain the necessary range of γ .

The average energy of cosmic ray particles measured at the earth is about 7 GeV .⁽⁴⁾ If one inspects Figs. II, III, IV, V and VI at \bar{T} of 6 or 7 GeV , the above remarks are more easily grasped.

Let us consider a particle of $m = 50 \text{ GeV}/c^2$ and $Z = 25$. When we solve Equation (33), we obtain the results shown in Fig. VII. The main differences from the previous results are that P_0 must increase into the hundreds of GeV/c range (with $P_m \sim 1 \text{ GeV}/c$), and that $|q|$ must be larger than before. The increase in P_0 is probably due to the Z^2 dependence of dE/dx . The larger value of $|q|$ implies that only the more rapid expansion is effective in producing the required fluctuations. Application of the above methods to different values of m give the same conclusions.

VI. Discussion and Conclusion

We have given a condensed account of the fluctuation origin of cosmic radiation. The pertinent results have been considered to find what requirements are placed upon the model. The assumption that the acceleration moment, a , is really dominated by deceleration, and hence is negative, implies that fluctuations may play a very important part in the origin of cosmic radiation. The requirement that $a = \text{constant } b$, where b is the fluctuation moment in the momentum changing processes, allows us to place certain restrictions upon the turbulent region in which cosmic ray particles experience acceleration and deceleration. We found that,

- a) for a fixed expansion velocity of the turbulent region, that
the number of scattering centers required decreases as the average velocity of the scattering centers increases;
- b) for a fixed number of scattering centers, the required average velocity of the scattering centers increases for increasing expansion velocity within the turbulent region.

The condition stated in a) is rather self-evident. The statement b) is a manifestation of the dominance of deceleration over an acceleration process. This gives rise to the importance of fluctuations. We point out that one can apply this requirement to any turbulent region in which deceleration is stronger than acceleration. It is then shown that this could be true in novae and supernovae shells.

We have shown that it is possible to obtain a power-law dependence upon the average kinetic energy for the number of particles ejected from a turbulent region. The restrictions imposed by the need for dominance of fluctuations places restrictions upon the rate at which the volume

is changing. In particular, when the total energy, E , is related to volume, V , by

$$E = \text{constant } V^{\frac{2}{3}}$$

we find that

$$-\frac{\gamma(1+a_1+a_2)}{\gamma(a_1+a_2)} < q < -\frac{\gamma}{q},$$

where $E_{\text{turb.}} = a_1 E_{\text{cr}}$ and $E_{\text{mag.}} = a_2 E_{\text{cr}}$. If one compares the expression for the total number of particles as a function of the average kinetic energy with the results from the fluctuation origin, it is possible to obtain further information about source requirements. We found that;

- a) when one considers protons, that the injection momentum, P_o , is normally bounded by $.1 \text{ GeV/c} < P_o < \text{approximately } 5\text{GeV/c}$;
- b) as greater energy is put into the turbulent or magnetic mode at the expense of the other two modes, that the chance for fluctuation origin to occur increases*;
- c) if the cosmic ray mode has much more energy than the other two modes, that a fluctuation origin becomes unlikely.
- d) the value of P_o for heavier particles goes approximately as Z^2 .

Much of the above analysis assumes that the turbulent region is expanding. There are many occurrences in astronomical phenomena where this is true. We would like to point out that this is not the only case where the fluctuation origin of cosmic rays may apply. The main assumption is that deceleration is stronger than acceleration in the turbulent region.

*This is true in the sense of a wider variation of γ (at a fixed \bar{T}) for a smaller change in P_o .

APPENDIX

We expect a power law solution in p . This suggests that we take the Mellin transform with respect to p of Eqn. (1) after first inserting Eqns. (7) and (8). We find that

$$(A1) \quad \frac{\partial g_1}{\partial t} - D_1 \nabla^2 g_1 - \left[(s-1)a + (s-1)(s-2)b - \frac{1}{T_1} \right] g_1 = 0$$

where

$$g_1 = \int_0^\infty p^{s-1} n_1(p, \underline{r}, t) d p.$$

If we write $g_1(s, \underline{r}, t) = f(\underline{r}, s, t) h(s, t)$ we can obtain

$$(A2) \quad \frac{\partial f}{\partial t} - D_1 \nabla^2 f = -\alpha^2 f,$$

$$(A3) \quad \frac{\partial h}{\partial t} = -\beta^2 h,$$

where

$$(A4) \quad \alpha^2 + \beta^2 = -\left[(s-1)a + (s-1)(s-2)b \right] + \frac{1}{T_1}.$$

The solution to Eqn. (A3) is

$$(A5) \quad h = e^{-\beta^2 t}.$$

Let $f = v(\underline{r}, t) e^{-\alpha^2 t}$; then we find

$$(A6) \quad \frac{\partial v}{\partial t} - D_1 \nabla^2 v = 0.$$

As we noted above, for particles in region I, the boundary condition is given by

$$(A7) \quad \frac{\partial v}{\partial x} + h v = 0.$$

The solution for v is ⁽¹⁰⁾

$$(A8) \quad v = \frac{q_0 P_0^{s-1}}{2\pi a_s R R_0} \sum_{n=1}^{\infty} \frac{(a_s h - 1)^2 + a_s^2 \alpha_n^2}{a_s^2 \alpha_n^2 + a_s h (a_s h - 1)} \sin \alpha_n R \sin \alpha_n R_0 e^{-D_1 \alpha_n^2 t}$$

where α_n are the positive roots of

$$(A9) \quad a_s \alpha \cot a_s \alpha + (a_s h - 1) = 0.$$

We will want $(A_s h - 1)$ to be large enough for a reasonable confinement time to allow for acceleration and fluctuation processes. If we then allow $r_0 \rightarrow 0$ (i.e., become a point source) and only consider the fundamental mode ($n = 1$; the exponential term allows this without introducing a serious error),

$$(A10) \quad v = \frac{q_0 P_0^{s-1}}{2 a_s^2 R} \sin \frac{3.1 R}{a_s} e^{-\frac{9.6 D_1}{a_s^2} t}$$

Thus we find that

$$(A11) \quad g_1 = \frac{q_0 P_0^{s-1}}{2 a_s^2 R} \sin \frac{3.1 R}{a_s} e^{-(\alpha^2 + \beta^2) t - \frac{9.6 D_1}{a_s^2} t}.$$

The solution for g_2 is given by

$$(A12) \quad g_2 = \frac{\beta}{r} \exp(-a_0^2 D_2 t - b_0 r),$$

where

$$(A13) \quad a_0^2 + b_0^2 = \frac{1}{D_2 T_2}.$$

When we apply the boundary conditions we have

$$(A14) \quad g_2 = \frac{1.55 q_0 \rho_0^{s-1}}{h a_s^3 r} \exp \left[- \left(\frac{1}{a_s} - \frac{h D_1}{D_2} \right) (r - a_s) - (\alpha^2 + \beta^2) t - \frac{9.6 D_1}{a_s^2} t \right].$$

The inverse Mellin transformation can be approximated by the method of steepest descent. This gives the asymptotic solution

$$(A15) \quad m_2 = \frac{1.55 q_0}{h \rho_0 a_s^3 r} \exp A,$$

where

$$A = -S_0 \ln(\rho/\rho_0) + [(S_0-1)a + (S_0-1)(S_0-2)b]t - \left(\frac{1}{a_s} - \frac{h D_1}{D_2} \right) (r - a_s) - \frac{9.6 D_1}{a_s^2} t - \frac{t}{T_1},$$

and

$$S_0 = \frac{1}{2b^2} \ln(\rho/\rho_0) + \frac{3b-a}{2b}.$$

The time average solution can be found by Laplace integration. The asymptotic solution is Eqn. (9).

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FIGURE CAPTIONS

- Fig. I. The variation of \bar{B} with the number of scattering centers at various values of Be . Similar results hold for different values of k .
- Fig. II. The variation of γ with \bar{T} for different P_o and P_m for $a_1=a_2=1$.
- Fig. III. The variation of γ with \bar{T} for different P_o and P_m for $a_1=2, a_2=1$.
- Fig. IV. The variation of γ with \bar{T} for different P_o and P_m for $a_1=10, a_2=1$.
- Fig. V. The variation of γ with \bar{T} for different P_o and P_m for $a_1=30, a_2=1$.
- Fig. VI. The variation of γ with \bar{T} for different P_o and P_m for $a_1=a_2=0.1$.
- Fig. VII. The variation of γ with \bar{T} for different P_o and P_m for $a_1=a_2=1$ and $m = 50 \text{ GeV}/c^2$.

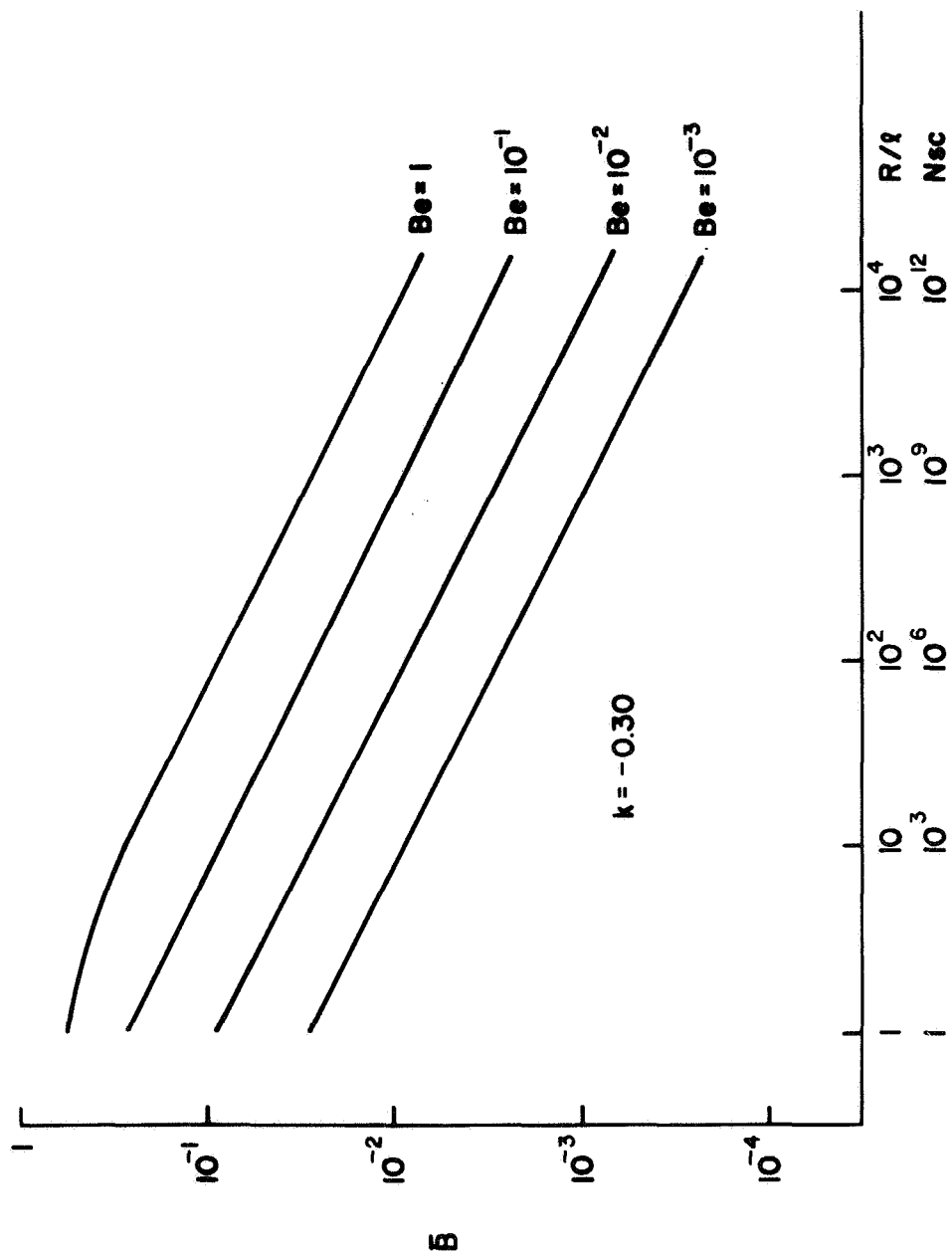


Fig. 1

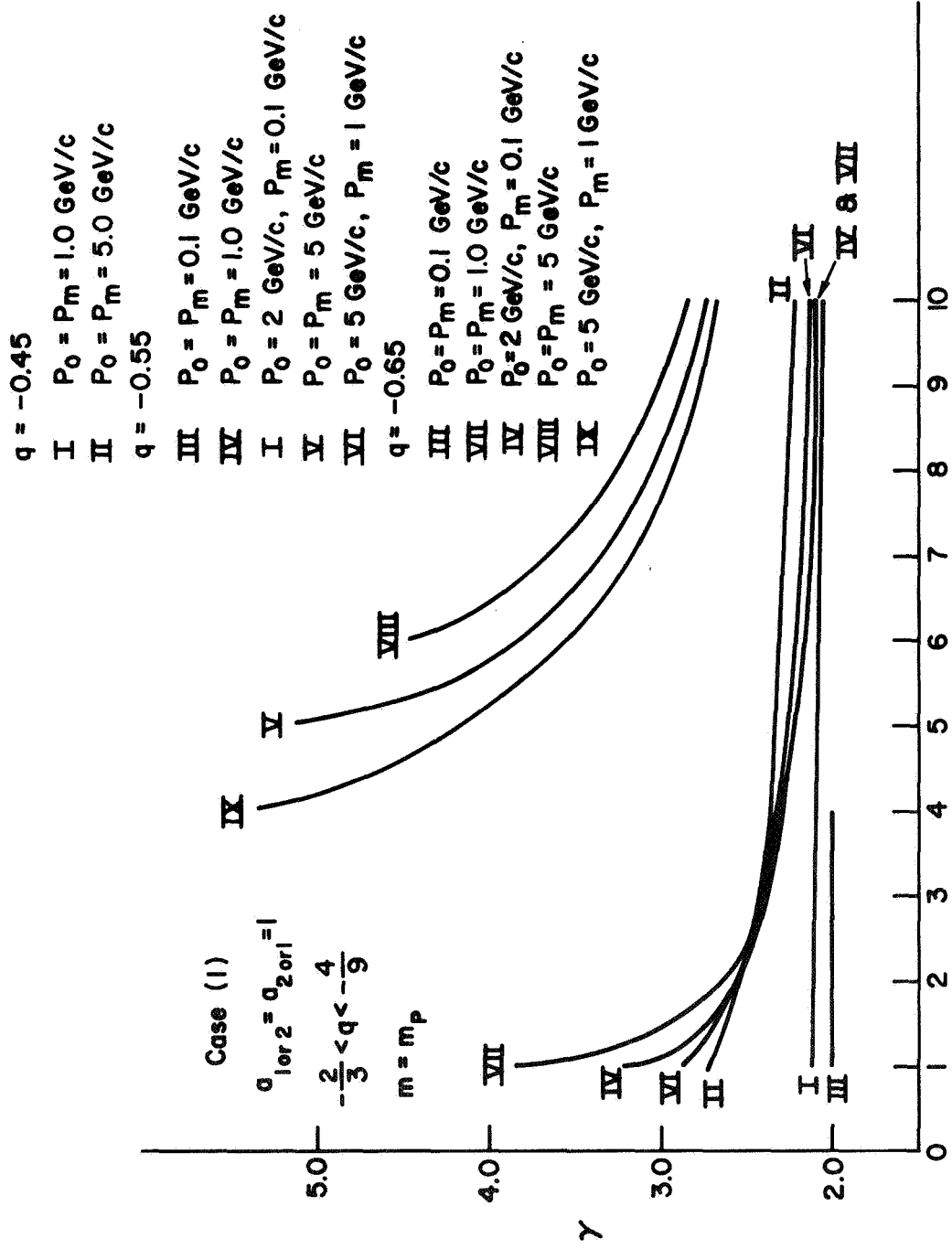


Fig. II

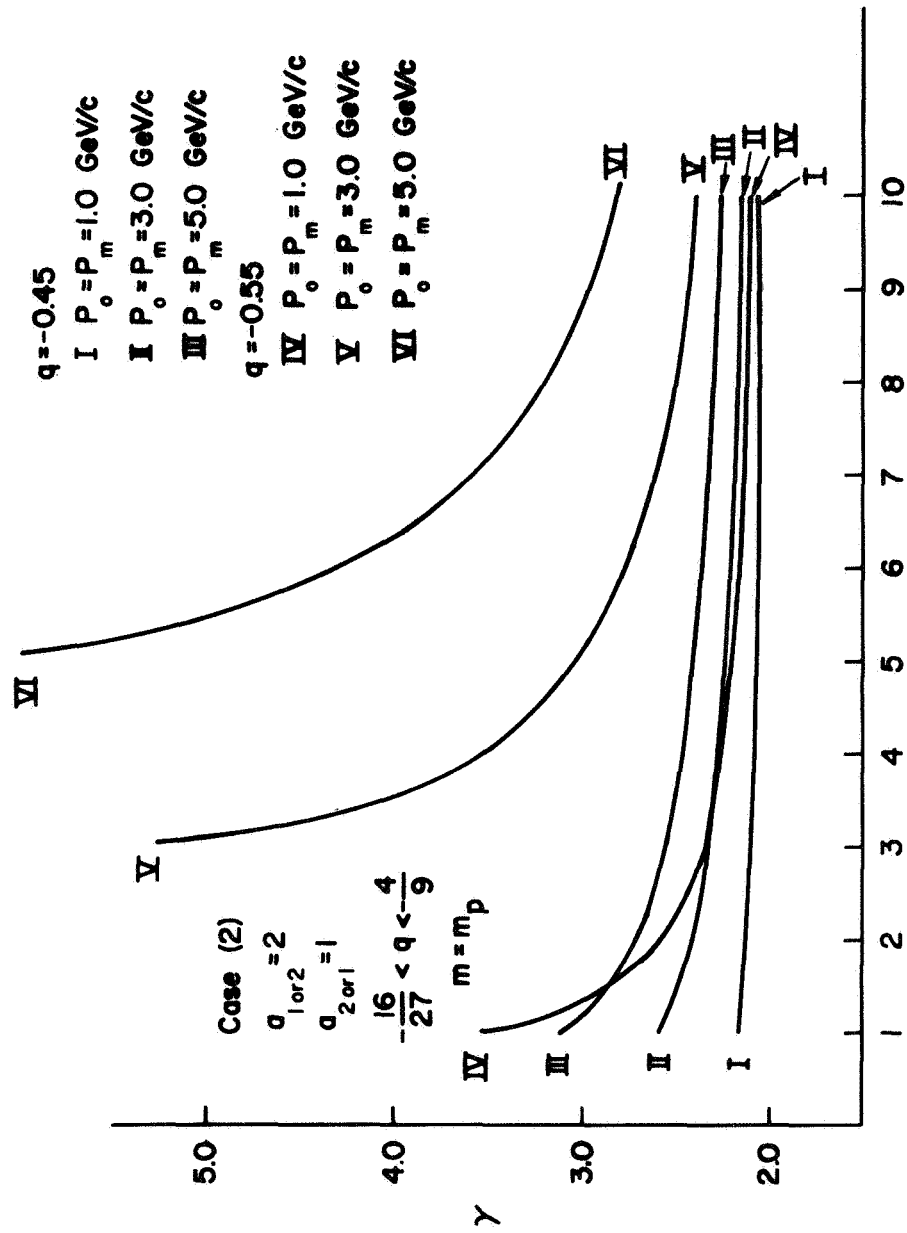


Fig. III

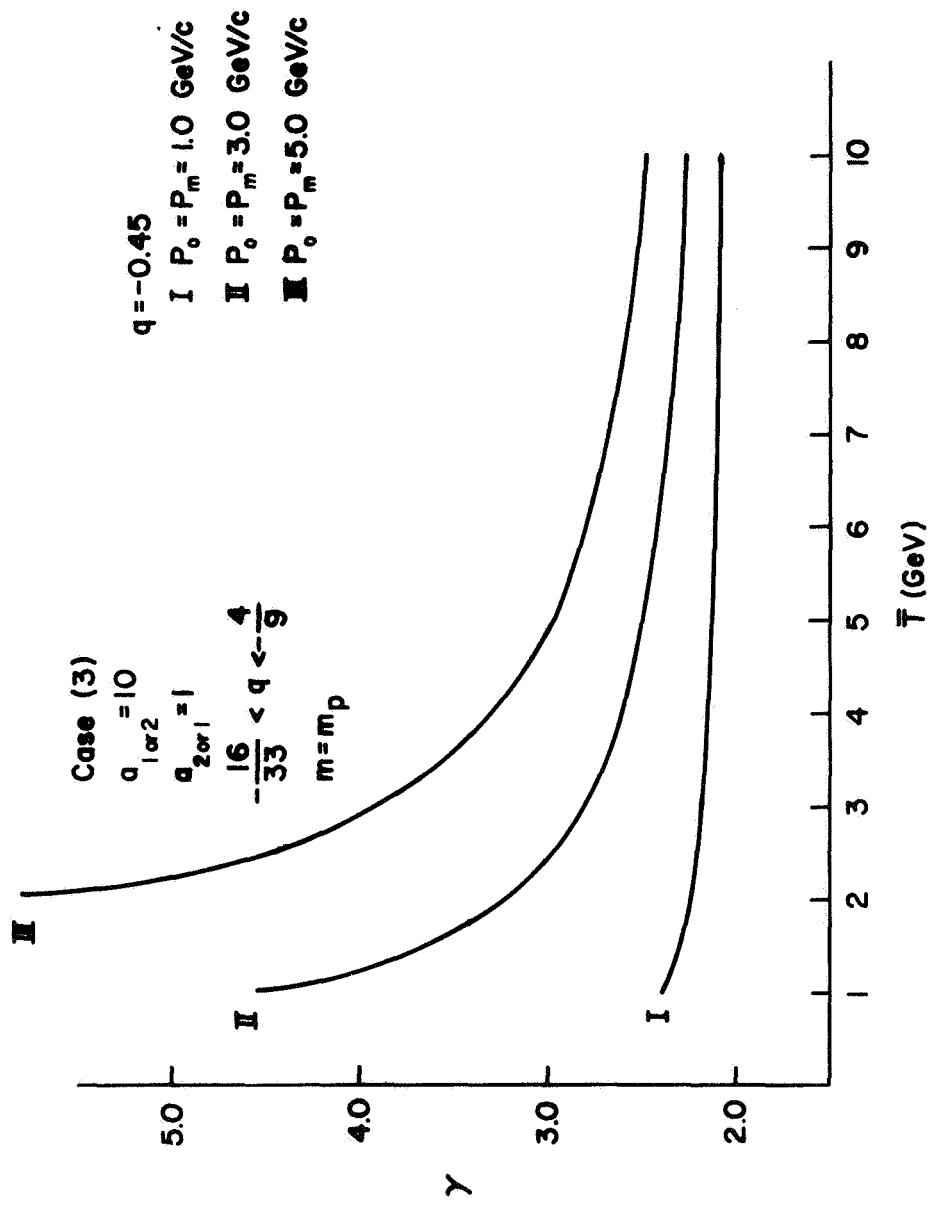
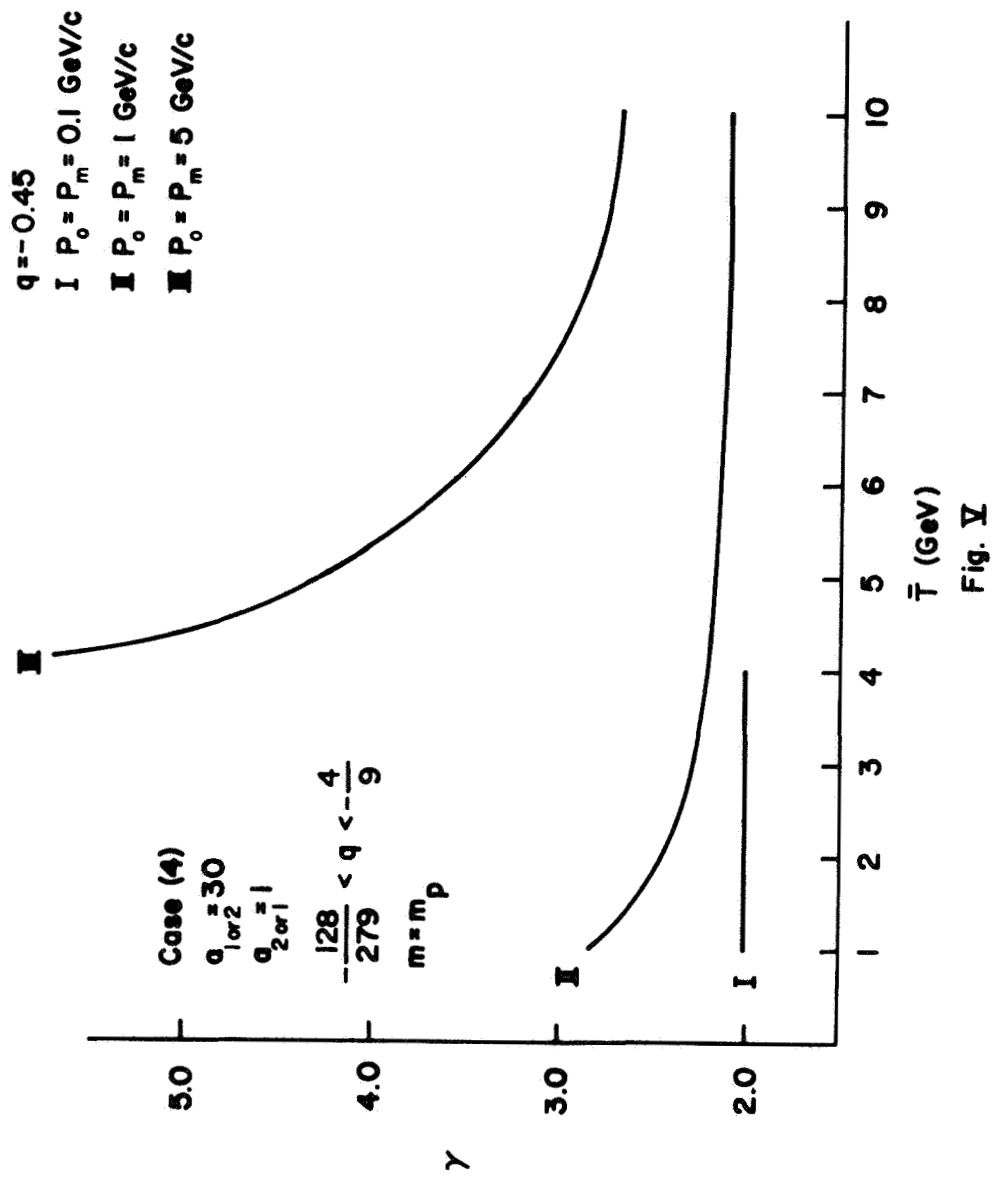
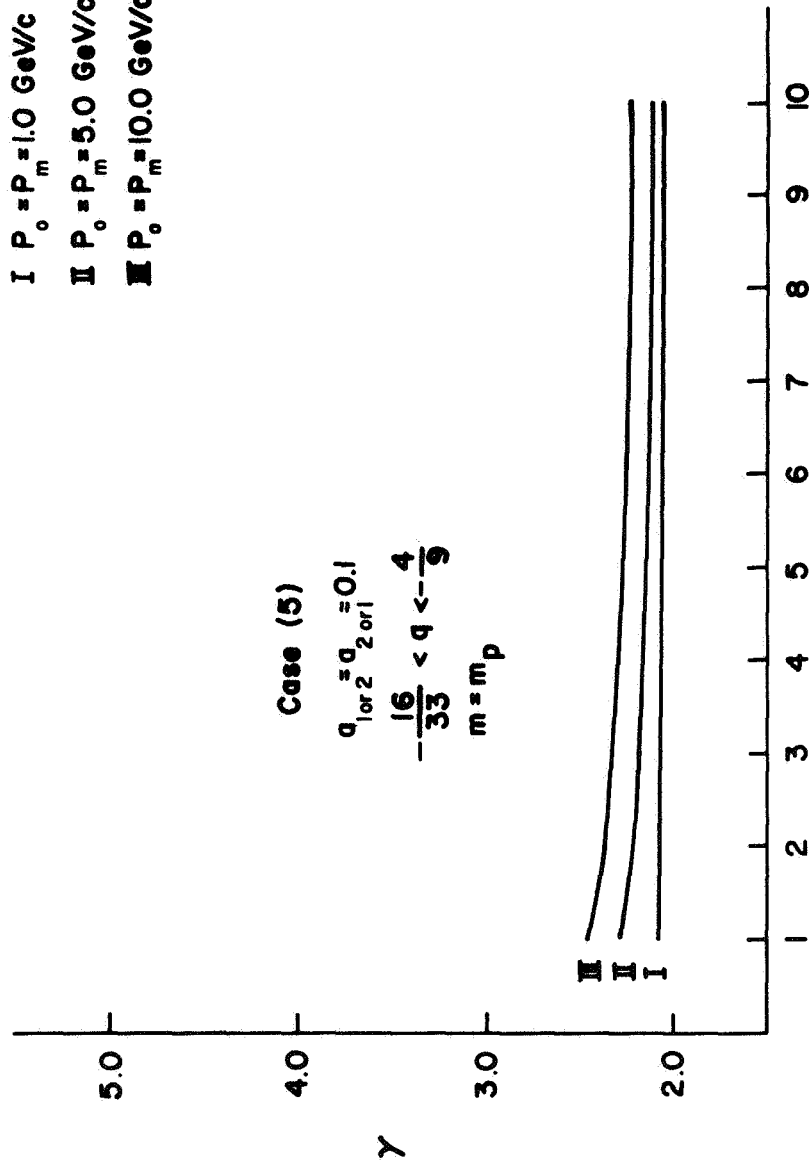


Fig. IV



$q = -0.45$
 I $P_0 = P_m = 1.0 \text{ GeV/c}$
 II $P_0 = P_m = 5.0 \text{ GeV/c}$
 III $P_0 = P_m = 10.0 \text{ GeV/c}$



Case (5)

$$q_{1 \text{ or } 2} = a_2 = 0.1$$

$$-\frac{16}{33} < q < -\frac{4}{9}$$

$$m = m_p$$

\bar{T} (GeV)

Fig. VI

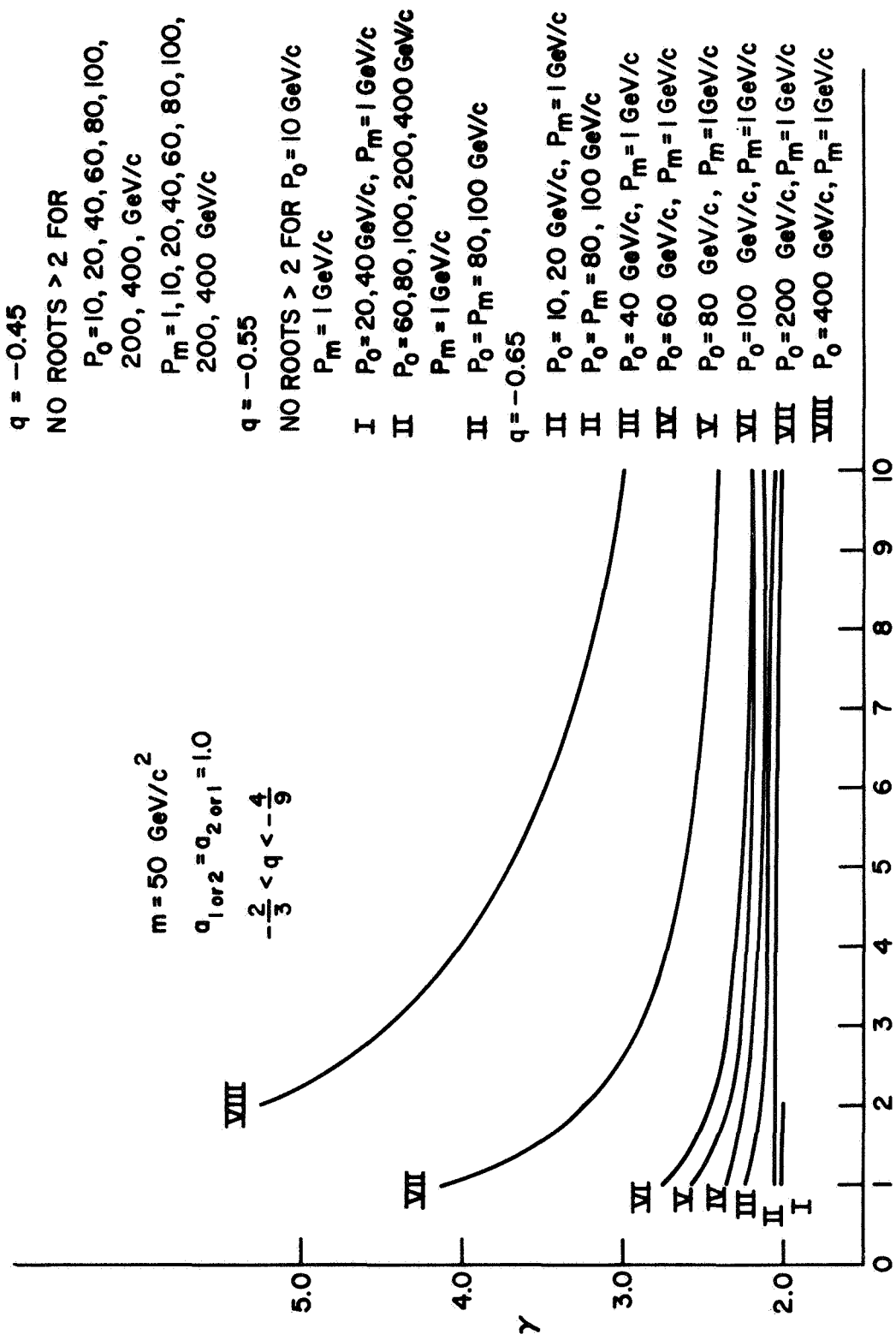


Fig. VII